FIRST SEMESTER MSc MATHEMATICS

ASSIGNMENT QUESTIONS (2022 ADMISSION)

LINEAR ALGEBRA COURSE CODE:MM211

QUESTIONS

1. Prove that the complex vector space over R is finite dimensional and find its basis

2. Prove that $KerT$ is a subspace of V for $T \in L(V, W)$

3. Find the m(T) for $T \in L(F^2, F^3)$ defined by

$T(x, y) = (x + y, x - y, 5x + 2y)$ with standard basis.

4. If $N \in L(V)$ is nilpotent then show that $N_1, N(v_1), \dots . v_1^{m(v_1)} \dots \dots . v_k, N(_{v_k}), \dots \dots . . N(v_{k)}^{m(v_k)}$ is a basis of V

5. Find $m(v_1)$ of N_1 for $N_1(z_1, z_2, z_3, \dots \dots . z_n) = (0, z_1, z_2, z_3, z_4, \dots \dots . z_{n-1})$.for $v_1 = (1,0,0,0,0)$ and $N_2(z_1, z_2, z_3, z_4, z_5) = (0, z_1, z_2, 0, z_4)$

6. Show that $mT(u_1, u_2 \dots \dots . . u_n) = A^{-1}MT(v_1, v_2 \dots . . v_n)A$

7. Find the Eigen values of T∈ L(F) defined by T (w, z) = (-z ,w) when (i) F= R (ii) F = C.

Differential Equations Course code:MM213

1. Find the general solution of the equation $y'' - 2y' + 5y = 25x^2 + 12$.
2. Find a function on $-1 \leq x \leq 1, 0 \leq y \leq 1$ which does not satisfy a Lipschitz condition.
3. Consider the differential equation $y' = 2xy$ and find a power series expansion$\sum a_n x^n$.
4. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of the power series in x.
5. In the differential equation $x^3(x-1)y'' - 2x(x-1)y' + 3xy = 0$ locate and classify the singular points on the x axis.
6. Detremine the nature of the point x=0 for the differential equation $xy'' + (sinx)y = 0$
7. Verify that $sin^{-1}(x) = xF(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2)$.
8. Transform the Chebyshev's equation $(1 - x^2)y'' - xy' + p^2y = 0$ into a hypergeometric equation by replacing x by $t = \frac{1}{2}(1 - x)$ and show that its general solution near x=1 is $y = c_1F(p, -p, \frac{1}{2}, \frac{1-x}{2})$.
9. Determine the nature of the point $x = \infty$ for Legendre's equation $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$
10. Show that

    i) $\frac{d}{dx}[J_0(x)] = -J_1(x)$

    ii) $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$

REAL ANALYSIS –I course code:MM212

Questions.

1) Determine which of the following functions are bounded variation on [0,1]

   a) $f(x) = x\sin\left(\frac{1}{x}\right)\ if x \neq 0\ and\ 0\ if\ x = 0$

   b) $f(x) = \sqrt{x}\sin x\,, if x \neq 0\ and\ 0\ if\ x = 0$

2).Give an example of a function which is not Riemann integrable but Stieljesintegrable.

3).If $f_n \to f$ uniformly and f_n is bounded on a set S. prove that $\{f_n\}$ is uniformly bounded on S.

4) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} x+y & if\ x \neq y \\ 1 & if\ x = y \end{cases}$.

Prove that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exists.

5). Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^2+y^2 & if\ both\ x\ and\ y\ are\ rationals \\ 0 & Otherwise \end{cases}.$$

Determine the points of $\mathbb{R}^2$ which $f_x\ and\ f_y$ exists.

6)Show that $f_{n(x)=\left\{\frac{1}{nx+1}\quad ,0<x<1, n=1,2,3,\ldots\ldots\ldots\right\}}$pointwise converges but not uniformily converges.

TOPOLOGY – I Course code: MM214

Questions.

1. Show that the set C of all complex numbers is a metric space with respect to the metric d, defined by

   $d(z_1 , z_2) = \frac{|z_1 - z_2|}{[(1+|z_1|^2)(1+|z_2|^2]^{\frac{1}{2}}}$ for all z_1 , z_2 in C.
2. Prove that A metric subspace (Y, d) of a complete metric space (X, d) is complete iff Y is closed.
3. Let E be a totally bounded subset of a metric space X. Show that every sequence { a_n} in E contains a Cauchy subsequence.
4. Let T be the class of subsets of N consisting of ∅ and all subsets of N of the form E_n ={ n , n +1 , n+2 , ….. } with $n \in N$.
   i) Show that T is a Topology on N
   ii) List the open sets containing the positive integer 6
5. Prove that a Topological space is compact iff every family of closed sets 7with empty intersection has a finite subfamily with empty intersection
6. Prove that every infinite subset of the Topological space has a limit point.
7. Prove that every compact Hausdorff space is a T_4 –space.