First semester M.Sc. Maths (2023 Admission)

Assignment Questions

Linear Algebra

1 Define the following with examples

i) Finite dimensional vector space ii) A sub space of an infinite dimensional vector space

2)$S_1 = \{(x_{1,} x_{2,}, x_{3,}) \in F^3,\ x_{1,} + 2x_{2,} + 3x_{3,} = 0\}$is a subspace

3) Let U and W are subspace of $R^8$with dimU=3 and with dimW=5 and dimU+dimW=8, Prove that $U \cap W = \{0\}$

4) Prove that if U and W are subspace of V then $U \cup W$ is asubspace of V iff one of the subspace is contained in the other

5) Prove that $T \in L(V)$,V is a finite dimensional then the following are equivalent a)T is invertible b) T is injective c)T is surjective

6) Prove that $dimL(V,W) = dimV.dimW$,What is $dimL(V,F)$,

7) Define Nilpotent operator with an example,

8) Find the basis of V if v_1 , $v_1, v_2 \dots\dots v_k \in V$ $with\, m(v_k) > 1$

9) Prove that $detAB = detA.detB$

10) Prove that $MT(u_1, u_1 \dots\dots u_n) = A^{-1}MT(v_1 v_2 \dots\dots v_n)A$

11) Define i)Trace ii) determinant iii)Eigen vector of linear operators

12) State and prove Cayley Hamilton

REAL ANALYSIS -1

Questions

1. Define the following with examples
   a) Bounded variation
   b) Total variation
   c) Path
   d) Rectifiable curve

e) Norm of a partition

j) step function

k) uniform convergence and continuity

2. $v_{f[a\,b]} = v_{f_1}[a\ b] + v_{f_2}[a\ b] + v_{f_3}[a\ b] + \cdots v_{f_n}[a\ b]$

3. Prove that $\int_a^b f d\alpha(s) ds = \int_a^b f\alpha' dt$

4. state and prove Euler's summation formula

5. State and prove Riemann's conditions for integrability with respect to γ

6. Define concavity and convexity.

7. Prove that $f(x,y) = \frac{x^2y}{x^4+y^2}$; $for\ (x,y) \neq 0$ is not continuous at (0,0).

8. Find the partial derivatives of the following functions

i) $f(x,y) = x^2 + y^2$

ii) $f(x,y) = \frac{xy}{x^2+y^2}$ if (x,y) $\neq$ (0 0)

= 0 if (x, y) = (0 0)

Differential Equations

1. Find the general solution of the equation $y'' - 2y' + 5y = 25x^2 + 12$.
2. Find a function on $-1 \leq x \leq 1, 0 \leq y \leq 1$ which does not satisfy a Lipschitz condition.
3. Consider the differential equation $y' = 2xy$ and find a power series expansion$\sum a_n x^n$.
4. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of the power series in x.
5. In the differential equation $x^3(x-1)y'' - 2x(x-1)y' + 3xy = 0$ locate and classify the singular points on the x axis.

6. Detremine the nature of the point x=0 for the differential equation $xy'' + (sinx)y = 0$
7. Verify that $sin^{-1}(x) = xF(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2)$.
8. Transform the Chebyshev's equation $(1 - x^2)y'' - xy' + p^2y = 0$ into a hypergeometric equation by replacing x by $t = \frac{1}{2}(1 - x)$ and show that its general solution near x=1 is $y = c_1F(p, -p, \frac{1}{2}, \frac{1-x}{2})$.
9. Determine the nature of the point $x = \infty$ for Legendre's equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$
10. Show that

    i) $\frac{d}{dx}[J_0(x)] = -J_1(x)$

    ii) $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$

TOPOLOGY – I

Questions.

1. Show that the set C of all complex numbers is a metric space with respect to the metric d, defined by

$$d(z_1, z_2) = \frac{|z_1 - z_2|}{[(1+|z_1|^2)(1+|z_2|^2]^{\frac{1}{2}}} \text{ for all } z_1, z_2 \text{ in C.}$$

2. Prove that A metric subspace (Y, d) of a complete metric space (X, d) is complete iff Y is closed.
3. Let E be a totally bounded subset of a metric space X. Show that every sequence { a_n} in E contains a Cauchy subsequence.
4. Let T be the class of subsets of N consisting of ∅ and all subsets of N of the form E_n ={ n , n +1 , n+2 , ….. } with $n \in N$.

    i) Show that T is a Topology on N

    ii) List the open sets containing the positive integer 6

5. Prove that a Topological space is compact iff every family of closed sets 7with empty intersection has a finite subfamily with empty intersection
6. Prove that every infinite subset of the Topological space has a limit point.
7. Prove that every compact Hausdorff space is a T_4 –space.

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