M. Sc. MATHS (FINAL)

2014 -2015

ASSIGNMENT TOPICS

School of Distance Education

UNIVERSITY OF KERALA

SENATE HOUSE CAMPUS, PALAYAM

THIRUVANANTHAPURAM – 695 034

Instructions

For continuous assessment you are requested to submit one assignment for each paper. The assignment must be neatly hand written and should contain not less than 15 pages with topic, introduction, sub-titles, conclusion, references/bibliography and page numbers. The attached format of facing sheet, duly filled in, should be submitted along with each assignment. Prepare the assignments with the help of standard reference book. Use your own sentences and please do not copy from text books or other assignments.

A total fee of **Rs. 100**/-should be paid in the university cash counter/ friends/ DD towards assignment fee and a photocopy of the same should be attached along with the assignments. The original receipt should be submitted to the AC II section of SDE.

The assignment must be submitted to **Dr. K. S. Zeenath, Co-ordinator , MSc Maths, School of Distance Education(SDE), University of Kerala**. Last date for submitting F**inal year assignments is 30th March 2015** . Those who submit their assignments after the specified dates have to pay a late fee to the university.

After valuation of the assignments, the internal marks will be published in the official website of SDE for 15 days for student scrutiny. Complaints regarding the internal marks must be communicated to the co-ordinator within these fifteen days. It is not possible to change the assignment marks once the mark list has been submitted to the University.

COMPLEX ANALYSIS I

1. Evaluate the integral $\int_C \frac{5z-2}{z(z-1)}dz$

2. Find the residue at $z=0$ for the following functions

   (i) $\frac{1}{z+z^2}$ (ii) $\frac{\cot z}{z^4}$ (iii) $\frac{\sinh z}{z^4(1-z^2)}$

3. Find the (i) $\int_C \frac{dz}{z^3(z+4)}$ (ii) $\int \frac{\cosh \pi z}{z(z^2+1)}$

4. Evaluate $\int_0^\infty \frac{x^{-a}}{x+1}x$ for $0<a<1$

5. Evaluate the following

   (i) $\int_0^{2\pi} \frac{\cos^2 3\theta}{5-4\cos 2\theta}d\theta$ (ii) $\int_0^{\pi} \sin 2^n \theta d\theta; n=1,2,....$

6. Find $\int_r z^n(z-1)^{-1}dz$ when $\gamma(t)=1\neq e^{2\pi i t}, 0\le t\le 1$ and $n\ge 1$

7. Using complex method prove that

   (i) $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta}=\frac{2\pi}{\sqrt{1-a^2}}$

8. Find the radius of convergence of

   (i) $\sum \frac{(z-1)^n}{2^n}$ (ii) $\sum n(n!)z^{n-1}$

COMPLEX ANALYSIS II

1. Discuss the nature of the following

(i) $f(z) = \dfrac{\cos z - 1}{z}$

(ii) $f(z) = \dfrac{(z^2 - 1)(z-2)^3}{\sin \pi z}$

(iii) $\dfrac{z}{e^z - 1}$ (iv) $\dfrac{z^2}{z^3 + z + 1}$ (v) $\dfrac{1}{1+z}$

2. Find the singular part of $f(z) = \dfrac{1}{(1+z^3)^2}$ at $z = -1$

3. Expand $e^{z+\frac{1}{z}}$ in Laurent series

4. Find the partial fraction expansion of $\dfrac{z+3}{z^3 - z^2 - 2z}$

5. Evaluate (i) $\int_{C_1} \sqrt{z}\,dz$ where $C_1 = z(t) = e^{it}\,\{0 \le t \le 2\pi\}$ and the value of $\sqrt{z}$ at $t = 0$ is 1

(ii) $\int_{C_2} \sqrt{z}\,dz$ when $C_2 = z(t) = e^{it}(-\pi \le t \le \pi)$, and the value of $\sqrt{z}$ at $t = -\pi$ is i.

6. Show that (a) $\lim\limits_{n \to \infty} \dfrac{n^2)n}{)n+z} = 1$

(b) $\Gamma'(1) + r = 0$

7. Show that $\dfrac{1}{)(z)}$ is an entire function of order 1.

FUNCTIONAL ANALYSIS I

1. Show that the following define norms on R^2 (i) $(x^2+y^2)^{1/2}$ (ii) $|x|+|y|$,

   (iii) $\left[|x|^P+|y|^P\right]^{1/P}$ with $p \geq 1$.

2. Show that $\sum \frac{|x_n|}{2^n}$ defines a norm on l_∞ and on L_1.

3. Prove that in R^n the following are norms.

   (i) $\|x\|_1 = \sum_{i=1}^{\infty} |x_i|$ (ii) $\|x\|_2 = \left[\sum |x_i|^2\right]^{1/2}$

   (iii) $\|x\|_\infty = \max_{1 \leq i \leq n} |x_i|$

4. Show that T is a bounded linear transformation on R^2, for $T(x,y) = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$

5. Show that the norm is continuous in the completion of a normed vector space.

FUNCTIONAL ANALYSIS II

1. Suppose $f \in L_2[-\pi,\pi]$. Show that $\int_{-\pi}^{\pi} f(t)\cos nt dt .... \to 0$ as $n \to \infty$

2. If T_1 and T_2 are normal operators and if U is an operator on a Hilbert space H such that $T_1U = UT_2$, show that $T_1^*U + UT_2^*$

3. Let X be a Hilbert space and $\psi : X \times X \to K$ be a sesquilinear functional. Let $q : X \to C$ be the associated quadratic form ie., $q(x) = \psi(x,x)$

   Prove that $4\psi(x,y) = q(x+y) - q(x-y) + iq(x+iy) - iq(x-iy)$

   b. ψ is symmetric iff q is real valued.

4. Let $H = l^2$ and $u_n = (0,0,1,0,...)$ then show that $\{u_1, u_2, ...\}$ is an orthonormal set in H .

5. $H = l^2[-1,1]$ and u_n is legender polinomial of degree n, show that $\{u_0, u_1, ..., u_n, ...\}$ orthonormal set.

OPERATIONS RESEARCH

1. Using Kuhn-Tucker solve minimize $f = (x_1 + 1)(x_2 - 2)$ over the region $0 \le x_1 \le x_2$; $0 \le x_2 \le x_1$

2. Solve by the method of quadratic programme Minimize

$-6x_1 + 2x_1^2 - 2x_1x_2 + 2x_1^2$ subject to $x_1 + x_2 \le 2, x_1 \ge 0; x_2 \ge 0$

3. Maximize $\sum_{n=1}^{4}(4u_n - nu_n^2)$ subject to $\sum_{n=1}^{4} u_n = 10$; $u_n \ge 0$

4. A cafeteria uses up paper napkins at the rate of 12 boxes per week. The space costs 20 cents per box per week the cost of placing an order is Rs.10.00. When the orders should be placed and for how many boxes should be placed.

5. List 10 types of problem that can be formulated and solved with dynamic programming what are the restrictions.

6. Solve the following problems using D.P.Maximise

$f(x) = 2x_1 - x_1^2 + x_2$ subject to $2x_1^2 + 3x_1^2 + \le 6$ $x_1, x_2 \ge 0$

7. Solve the following problem using D.P. Minimize

$f(x) = 2x_1^2 - 3x_2 - 4x_3^3$ subject to

$3x_1 + 2x_2 + 6x_3 \ge 16$; $x_2 \le 4$; $x_3 \le 5$; $x_1, x_2, x_3 \ge 0$

8. If a factory maintains an average in process in inventory equivalent to 300 work orders, or jobs, and an average job spends 6 weeks in the factory, what the production rate of the factory in units of jobs per year.

9. Consider a barber shop with two chairs and two barbers and no room to wait. On the average a customer arrives every 10 minutes and each haircut takes an average of 15 minutes.

(i) Calculate the expected number of busy barbers.

(ii) Calculate the expected number of customers turned away per hour.

(iii) Set up the transition diagram and steady-state equations.

(iv) Solve the steady-state equation for the distribution of the number of customers in the shop.

10. For the following data how should a student should study to maximum grade

	A	B	C
0	1	1	0
1	2	1	1
2	2	3	3
3	4	3	4

11. Minimize $f = (x_1 - 2)^2 + x_2^2$ subject to $x_1^2 + x_2 - 1 \leq 0$; $x_1 \geq 0$; $x_2 \geq 0$

Obtain the solution graphically.Write the Kuhn-Tucker conditions.

12. Use K.T.Conditions find the minima and maxima of

$(x_1 - 4)^2 + (x_2 - 3)^2$ subject to $36(x_1 - 2)^2 + (x_2 - 3)^2 \leq 9$

13. An item is produced at the rate of 50 items per day. The demand occurs at the rate 25 items per day. If the set up cost is Rs.100.00 and holding cost is Rs.0.01 per unit of item per day, find EOQ assuming that the shortage are not permitted. Also find the time of cycle and minimum total cost for one run.

14. The following table lists the jobs of a network along with their lime estimates.

job		Duration days		
i	j	Optimistic	Most likely	Pessimistic
1	2	3	6	15
1	6	2	5	14
2	3	6	12	30

2	4	2	5	8
3	5	5	11	17
4	5	3	6	15
6	7	3	9	27
5	8	1	4	7
7	8	4	19	28

a. Draw the project network

b. Calculate the length and variance of critical path.

15. Minimize $f(x) = abc + \left(\frac{a^2 + b^2 + c^2}{2}\right)$ subject to

$g_1(x) = a + b + c - 1 \leq 0$

$g_2(x) = 4a + 2b - \frac{7}{3} \leq 0$

$a, b, c \geq 0$

16. Minimize $f(x) = -x_1 - x_2 - x_1 + \frac{1}{2}(x_1^2 + x_2^2 + x_3^3)$ subject to

$g_1(x) = x_1 + x_2 + x_3 - 1 \leq 0$

$g_2(x) = 4x_1 + 2x_2 - \frac{7}{3} \leq 0$

ANALYTIC NUMBER THEORY

1. (i) If $(a,b) = 1$ and if c / a and a / b then prove that $(c,d) = 1$

(ii) If $(a,b) = 1$ and $d / a + b$, prove that $(a,d) = (b,d) = 1$

2. Given x and y, let $m = ax + by$; $n = cx + dy$ where $ad - bc \neq 1$. Prove that $(m,n) = (x,y)$

3. Prove that $n^4 + 4$ is composite of $n > 1$

4. Prove that every number of the form $2^{a-1}(2^a-1)$ is perfect if 2^a-1 is prime.

5. If $(a,b)=1$ show that

(i) $(a^n,b^k)=1$ for $n\geq 1$ and $k\geq 1$

(ii) $(a+b,a-b)$ is either 1 of 2

(iii) $(a+b,a^2-ab+b^2)$ is either 1 or 3.

6. If $m\neq n$, compute the gcd of (a^{2m+1},a^{2n+1}) in terms of a.

7. Let $d=(826,1890)$. Using Euclidean algorithm to compute d and express d as a 1.c of 826 & 1890.

8. Find all x which satisfy the system of congruencies

$x\equiv 1(\text{mod}\,3), x\equiv 2(\text{mod}\,4); x\equiv 3(\text{mod}\,5)$

9. Find all odd primes p for which 5 is a quadratic residue.

10. Prove that $\sum_{d/n}\mu(d)=\mu^2(n)$

12. Find all integers n such that

a. $\varphi(n)=\frac{n}{2}$

b. $\varphi(n)=\varphi(2n)$

c. $\varphi(n)=12$

12. Prove that $5n^3+7n^5\equiv 0(\text{mod}\,12)$ for all integers.

13. Find all positive integers n for which $n^{13}=n(\text{mod}\,1365); n^{17}=n(\text{mod}\,4080)$

14. Show that the Diophantine equation $y^2=x^3+11$ has no solution.

15. find (219/383)

16. Solve $25x\equiv 15(\text{mod}\,120)$

GROUP REPRESENTATION THEORY

1. Find a right regular representation of the group $G = \{1, w, w^2, w^3 / w^4 = 1\}$

2. Show that the map $a \to C(a) = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ define a representation of $gp\{< a > / a^3 = 1$.

   Prove that it is irreducible over the field of real numbers.

3. Show that the permutation representation of S_3 induced by the subgroup

   $H = 1, (12)$ is isomorphic to S_3.

4. Let $B(x)$ be a representation of the cyclic group $G = \{1, a, a^2 / a^3 - 1\}$ given by

   $B(a) = \begin{bmatrix} 1 & 0 \\ 1 & w \end{bmatrix}$ when $w^3 = 1$. Find an invertible matrix T such that $T^{-1}B(x)T$ is in

   diagonal form for all $x \in G$.

5. Let $G = \{1, a, b, c\}$ be the klien 4 group and a representation of G is given by

   $A(a) = A(b) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $A(c) = 1$. Find $C(A)$ and centre of $C(A)$.

6. Find the character table for D_4.

7. Let $G = \{1, w, w^2, / w^3 = 1\}$ and define

   $$A(1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; A(w) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

   $A(w^2) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ show that this is a representation of G.

8. Find the Fourier expansion of the character ζ of the right regular representation of S_3.

9. Find Fourier expansion ϑ of the representation in example (3).

10. Let $G = S_3$ and $H = \{1, a\}$ be a subgroup of G where $a = (1,2)$. Let φ be a linear

    character of H for which $\varphi(a) = -1$. Find the induced character φ^G.

11. Verify whether S_3, S_4, A_4 and D_4 are Frobenius groups.

12. Show that $1+\frac{i}{\sqrt{2}}, 1+i\sqrt{2}$ are algebraic integers.

13. Show that A_4 has a normal subgroup of order 4.

14. Let $G = S_3$ and $H = (1, \rho_4, \rho_5)$ and $\phi(1) = 1, \phi(\rho_4) = w, \phi(\rho_5) = w^2$. Find the induced character ϕ^G .

15. Write the character table of $S_4, Z_5, Z_5 \times Z_3 + Z_3 \,\&\, Z_7$

16. Show that S_4 is a Frobenius group.

NUMERICAL ANALYSIS WITH COMPUTER APPLICATIONS

1. Use the method of false position to obtain a root, correct to three decimal places, of each of the following.

(i) $x^3 + x^2 + x + 7 = 0$

(ii) $x^3 - x - 4 = 0$

(iii) $x^3 - x^2 - 1 = 0$

2. Use iteration method find correct to four significant figures.

(i) $\cos x = 3x - 1$

(ii) $x = \frac{1}{(x+1)^2}$

(iii) $\sin^2 x = x^2 - 1$

3. Use Newton-Raphson method obtain the root correct three places

(i) $x^3 - 5x + 3 = 0$

(ii) $x^4 + x^2 = 80 = 0$ $\qquad x^4 + x - 80 = 0$

4. Apply Graeffe's root-squaring method determine the approximate solution of $x^3 - 2x^2 - x + 2 = 0$

5. From the following values of x and y , find $\frac{dy}{dx}$ when $x = 6$

x	y
4.5	9.69
5	12.90
5.5	16.71
6	21.18
6.5	26.37
7.0	32.24
7.5	39.15

6. A function $y = f(x)$ is defined as follows

x	y
1.0	1.0
1.05	1.025
1.10	1.049
1.15	1.072
1.20	1.095
1.25	1.118
1.30	1.140

7. Obtain a root of each of the following equations correct to three decimal places using Bisection method and Method of false position

(a) $x^3 + x^2 + x + 7 = 0$

(b) $x^3 - 4x - 9 = 0$

(c) $x^3 - x^2 - 1 = 0$

8. Use Muller's method to find a root of the equations

(i) $x^3 - x - 1 = 0$

(ii) $x^3 - x^2 - x - 1 = 0$

9. Using Lagrange'sinterpolation formula, express the function

$\frac{x^2 + x - 3}{x^3 - 2x^2 - x + 2}$ as sums of partial fractions.

10. Evaluate $\int_1^3 \frac{1}{x}$ using Simpsins rule with 4 strips and 8 strips respectively.

11. Compute the values of

$I = \int_0^1 \frac{dx}{1 + x^2}$ using trapezoidal rule with $h = 0{,}5{,}0{,}25$

12. Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$, obtain the Tawylor series for $y(x)$ and compute $y(0{,}1)$ correct to four decimal places.

13. Using picards method, obtain the solution of $\frac{dy}{dx} = x(1 + x^3 y), y(0) = 3$ Tabulate the values of $y(0{,}1), y(0{,}2) \ldots y(1{,}0)$

14. Use Runge-Kutta fourth order method to find the value of y when $x = 1$, given that $y = 1$ when $x = 0$ and that $\frac{dy}{dx} = \frac{y - x}{y + x}$

15. Solve the boundary value problem

$\frac{d^2 y}{dx^2} + y = 0$; with $y(0) = 1$ and $y(0) = 0$. Compute $y(0{,}2)$ and $y(0{,}4)$

16. Find a root of the equation $x^3 - x - 1 = 0$ by bisection method.

17. Find a double root of the equation $f(x) = x^3 - x + 1 = 0$

18. Find all roots of the equation $x^3 - 9x^2 + 18x - 6 = 0$ by Graeffe method.

19. Find the cubic polynomial which takes following values

$y(0) = 1, y(1) = 0, y(2) = 1, y(2) = 10$. Also obtain $y(4)$

20. If $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$

Find $y(0,1)$ correct to four decimal places using Taylors series.

21. Find $y(0,8)$ and $y(1,0)$ using Milins Thamson method by solving $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0$.

22. Evaluate $\int_0^1 \int_0^1 e^{x+y} dx.dy$ using (1) Trapizoidal rule (ii) Simpson's rule.

23. Solve by factorization

$5x - 2y + z = 4$

$7x + y - 5x = 8$

$3x + 7y + 4x = 10$

24. Solve by Gauss Seidel Method

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10 = 22$

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M.Sc. MATHS (Final) Assignment 2014-15

Course : -------------------- (Previous/Final) ------------------------------

Year of admission: ---

Paper Code : ---

Title of the Paper : ---

Name and Address of the Contact Centre

SDE Enrolment Number

Exam Register Number (Compulsory)

Name and Address of the Student

Student's Name: ---

Address: ---

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/ 15

Name and Signature of the Evaluator