M. Sc. Mathematics

(Previous)

2014 -2015

ASSIGNMENT TOPICS

School of Distance Education

UNIVERSITY OF KERALA

SENATE HOUSE CAMPUS, PALAYAM

THIRUVANANTHAPURAM – 695 034

Phone: 0471 2300137

Instructions

1. All students are requested to submit one assignment for each paper.
2. The assignment must be neatly handwritten by the concerned student .For Computer Programming it should be practical record. Candidates are permitted to perform practical in any computer centre Algorithm should be handwritten and on the next page its computer programme with an output should be entered.
3. The downloaded facing sheet must be filled and attached with each assignment.
4. Write in your own sentences with the help of autherised books and do not copy from SDE study materials.
5. An assignment valuation fee of Rs. 100/- should be paid in the university cash counter/friends/DD as assignment fee .The photocopy of the receipt should be attached along with the assignments. The original receipt should be submit in the ACII section of School of Distance Education.
6. The mark list of assignments (Internal Marks/Continuous Assessment) will be published in the official website of SDE for 15 days after valuation for student scrutiny. Complaints regarding assignment marks should be communicated to the co-ordinator during these 15 days. It is not possible to change the final assignment marks once it is submitted to the University.
7. The assignment must be submitted to Dr. K.S. Zeenath, Co-ordinator , MSc Mathematics, School of Distance Education, University of Kerala on or before 30.06.2015. Those who submit their assignments after the specified date has to pay a late fee to the University.
8. Students are requested to write the correct Phone number in the facing sheet.

ASSIGNMENT QUESTIONS

LINEAR ALGEBRA

1. If $\alpha = (1,2,1), \beta = (3,1,5), \gamma = (3,-4,7)$. Prove that the sub spaces spanned by $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \gamma\}$ are same.

2. Find the co-ordinates of $(1,2,1)$ in R^3 in the ordered basis (1,0,1), (0,1,1) (1,1,0) of R^3 .

3. Find a basis for the subspace of R^4 spanned by the vectors (1,2,1,3) (2,1,1,3),

   (1,4,1,3) .

4. Find a linear operator on R^3, whose null space in spanned by (1,1,1) and (1,2,1). Is it invertible? Why?

5. Let T be the linear operator on R^3 given by $T(\varepsilon_1) = (2,0,0)$, $T(\varepsilon_2) = (0,2,0)$ and $T(\varepsilon_3) = (0,0,-1)$. Prove that has no cyclic vector. Find $z(\varepsilon_1, T)$.

6. Find $T(x_1, x_2, x_3)$ where $T(1,1,1) = 3$, $T(0,1,-2) = 1$, $T(0,0,1) = -2$.

7. Find the basis and dimension of Range and null space of

   $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3)$

8. Find the matrix representation of the following T on R^3 relative to the basis

   (a) $\beta_1 = \{(1,3)(2,5)\}$, $\beta_2 = \{(1,0)(0,1)\}$

   (i) $T(x, y) = (3x - 4y, x + 5y)$

9. Let T be the linear operator on R^4 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$. Under what conditions on a, b and c is T diagonalizable.

10. Let T be the linear operator on R^3 which is represented by $A=\begin{bmatrix}3 & 1 & -1\\ 2 & 2 & -1\\ 2 & 2 & 0\end{bmatrix}$. Prove that T is diagonalizable.

Paper II

REAL ANALYSIS I

1. Prove that the function ' f ' given by $f(x)=x^2\cos\left(\frac{1}{x}\right)$; for $x\neq 0$, $f(0)=0$ is of bound variation on [0,1]

2. Compute the positive, negative and total variation of $f(x)=[x]-x;0\le x\le 2$

3. Evaluate

   (i) $\int_0^2 x.d\alpha(x)$, where $\alpha(x)=\begin{cases} x;0\le x\le 1\\ 2+x;1<x\le 2\end{cases}$

   (ii) $\int_0^3 f(x)d([x]+x)$ where $f(x)=\begin{cases}[x];0\le x<\frac{3}{2}\\ e^x;\frac{3}{2}\le x\le 3\end{cases}$

4. Text the following sequence for uniform convergence

   (i) $\left\{\frac{\sin nx}{\sqrt{x}}\right\};0\le x\le 2\pi$

   (ii) $\left\{\frac{x}{n+x}\right\};0\le x\le k$

   (iii) $\left\{\frac{n^2x}{1+n^3x^2}\right\};0\le x\le 1$

   (iv) $\left\{\frac{nx}{1+n^3x^2}\right\};0\le x\le 1$

5. Show that $f_n(x)=\frac{\lg(1+nx^2)}{n^2}$ is uniformly convergent on the interval [0,1].

6. Test the convergence of $f_n(x)=\frac{n^2x}{1+n^3x^2}$ in [0,1]

7. Show that $f(x,y,z)=(x+y+z)^3-3(x+y+z)-24xyz+a^3$ has a minima at (1,1,1) and a maxima at (-1,-1,-1)

8. Show that the following functions have a minima at the points indicated

(i) $x^2+y^2+z^2+2xyz$ at (0,0,0)

(ii) $x^4+y^4+z^4-4xyz$ at (1,1,1)

Paper III

DIFFERENTIAL EQUATION

1. Find the general solution of $y''-2y'+2y=e^x \sin x$

2. Find a particular solution of $y''-y'-6y=e^{-x}$ using the method of variate.

3. Using picard's method of successive approximations find an exact solution of the initial value problem $y'=y; y(0)=1$.

4. Find the general solution of $(1+x^2)\ y''+2xy'-2y=0$ in terms of power series in x.

5. Solve $x^3y''+(\cos 2x-1)y'+2xy=0$

6. If $T_n(x)$ is the nth Chebyshev polynomial, derive $T_n(x)+T_{n-2}(x)=2xT_{n-1}(x)$

7. Prove that $\left(n-\frac{1}{2}\right)!=\dfrac{(2n)!\sqrt{\pi}}{2^{2n}.n!}$

8. Find the general integral of $z(xp-yq)=y^2-x^2$

9. Find the integral of $yzdx+2xzdy-3xydz=0$

10. Solve $p=(z+qy)^2$ by Jacobi's method.

Paper IV

TOPOLOGY I

1. Show that $d(x,y)=\begin{cases} 0 & if\ x=y \\ 1 & if\ x\neq y \end{cases}$ is a metric on X .

2. Let $X=\{a,b,c,d\}$ and $F=\{\varphi,\{a\},\{b\},\{ab\},X\}$

   Show that F is a topology for X .

T

3. Show that cantor's set T is closed.

4. Let S be the collections of subsets of N consisting of empty set φ and all subsets of the form $G_m = \{m+n+1,.....\} m \in N$

Show J is a topology for N

X

5. Let (X,d) be a metric space. Show that $d^* = X \times X \to R$ defined by $d^*(x,y) = \dfrac{d(x,y)}{1+d(x,y)}$ is also a metric in X and that d, and d^* are equivalent.

6. Show that every discrete space is Hausdorff.

7. Let $J = \{\varphi\{a\},\{c,d\},\{a,c,d\},\{b,c,d,e\}X\}$ on the set $X = \{a,b,c,d,e\}$ and show that

(i) (X,Y) is disconnected

(ii) $Y = \{b,d,e\}$ is a connected subset of X .

Paper V

ALGEBRA

1. Find the index of $<2>$ in Z_{24}

2. Find the order of (3,6,13,16) in $z_4 \times z_{12} \times z_{20} \times z_{24}$

3. Construct a field having 8 elements

4. Find the basis of $Q\left[\sqrt{2},\sqrt{3}\right]$ over Q

5. Find the sylow 2 subgroups of S_4 and show that they are conjugate.

6. Find isomorphic refinements of the series $\{0\} < 60z < 30z < z$ and $\{0\} < 60z < 20z < z$

7. Find the Galois group of the splitting field of $x^4 - 1$ over Q .

Paper VI

REAL ANALYSIS II

1. Prove the existence of uncountable of sets of measure zero.

2. If $m^*E = 0$ show that is measurable.

3. Show that $m^*A = m^*(A+x)$ where $A+x=\{y+x / y\in A\}$

4. Find Lebesgue integral

$f(x)=0$ if x is rational

=1 if x is irrational over [0,1]

5. Calculate Lebesgue integral for the function

$$f(x)=\begin{cases} 1 \ \ where\ x\ is\ rational \\ 2 \ \ when\ x\ is\ rational \end{cases}$$

6. Evaluate $\int_a^b f(x)dx$ if $f(x)=\begin{cases} 0;0\le x<1 \\ 1;(1\le x<2)\bigcup\{3\le x<4\} \\ 2;\{2\le x<3\}\bigcup\{4\le x<5\} \end{cases}$

7. Verify Bounded Convergence Theorem for the sequence of the function

$$f_n(x)=\frac{1}{\left(1+\frac{x}{n}\right)}n;0\le x\le 1;n\in N$$

Paper VII

TOPOLOGY II

1. Show that $[f_1]*[f_2]=[f_1*f_2]$ is a well defined operation in $\pi_1(X,x_0)$

2. Let $f_n : R\to R$ be the function

$$f_n(x)=\frac{1}{n^3[x-(1/n)]^2+1}$$

   a. Show that $f_n(x)\to f(x)$ for each $x\in R$

   b. Show that f_n doesnot converge uniformly to ‘ f ’.

3. Let $\sigma^2=\langle a_0,a_1,a_2\rangle$. Assign the order $a_0<a_1<a_2$ what are $+\sigma^2$ and $-\sigma^2$ what are their geometrical interpretation.

4. Show that $\pi_1(S^n,S_0)=0$ for $n>1$

5. Prove that if (X,x_0) and (Y,y_0) are based spaces then

$$\pi_1(X\times Y,(x_0,y_0))=\pi(X,x_0)\times\pi_1(y,y_0)$$

6. Prove that the map $e^{(-\frac{1}{2},\frac{1}{2}),(-\frac{1}{2},\frac{1}{2})} \to S'-\{-1\}$ is a homeomorphism.

7. Prove that $\deg \pi_1(S', S_0) \to Z$ is an isomorphism.

8. Prove that the function $h(x) = \dfrac{e^x}{1+e^x}$ is a homeomorphism from the real line to the open interval (0,1)

9. If X and Y are path connected spaces show that $\pi(X \times Y)$ is a isomorphic to $\pi_1(X) \times \pi_1(Y)$

10. Let α, β the paths in A defined by $\alpha(s) = (s+1,0)$ and $\beta(s) = h\alpha(s), 0 \le s \le 1$.

 Show that if h is homotopic to the identity relative to the two boundary circles of A then $\alpha^{-1}\beta$ is homotopic related {1,0} to the constant loop at the point (1,0).

11. Let $\pi : X \to Y$ be a covering map, $p \in Y, q \in \pi^{-1}(p)$ and $F : I \times I \to Y$ a map such that $F(0,t) = F(1,t) = p$ for $0 \le t \le 1$. Find a map $\overline{F} : I \times I \to X$ such that $\pi_0 \overline{F} = F$ and $\overline{F}(0,t) = q, 0 \le t \le 1$. Check that $\overline{F}$ is unique.

12. Prove that a set of $(k+1)$ points in R^n is geometrically independent iff no $(p+I)$ points lie in a hyperplane of dimension less than or equal to $p-1$.

13. Define the concept of incidence number. Let K be an oriented complex, σ^p an oriented $p-$ simplex of K and $\sigma^{p-2}, \sigma^{p-2}$ face of σ^p. Show that

$$\sum \left[\sigma^p, \sigma^{p-1}\right]\left[\sigma^{p-1}, \sigma^{p-2}\right] = 0, \sigma^{p-1} \in K$$

14. Compute second incidence matrix of $\sigma^3 = (a_0, a_1, a_2, a_3)$ where

 $a_0 = (0,0,0) a_1 (1,0,0) a_2 (0,1,0)$ in R^3 with $a_0 < a_1 < a_2 < a_3$

15. Show that the unit 2 sphere

 $S^2 = \{(x_1, x_2, x_3 \in R^3 / x_1^2 + x_2^2 + x_3^3 = 1\}$ is a triangulable.

16. Show that the unit circle $S' = \{(x_1, x_2) \in R^2 / x_1^2 + x_2^2 = 1\}$ is a deformation retract of the punctured plane $R^2 / \{0\}$. Deduce that $\pi_1\{R^2 / \{0 \cong Z\}$.

17. Show that there is a vector field on S_n $n \ge 1$ if and only if n is odd.

18. Show that $\pi_1(A, x_0) \cong \pi_1(X, x_0)$; where X is a deformation retract of a space and $x_0 \in A$.

19. Show that for any complex k, $\lim\limits_{s \to 0} meshk(x) = 0$ mesh $k(s) = 0$.

20. Compute all homology groups of K where K denotes complex consisting of all proper faces of a 2-simplex $< a_0, a_1, a_2 >$ with orientation induced by the order $a_0 < a_1 < a_2$.

Paper VIII

COMPUTER PROGRAMMING IN C^{++}

Programs

1. Usually a line is defined by two points a two dimensional plane. A point is defined by an x coordinate and a y coordinate. Write a programe to define a structure to represent the points and the program should calculate the length of the line, after the user enters the two points.

   Length of line $(x_2 - x_1)^2 + (y_2 - y_1)^2$

2. Create a structure date that contains three members: month, day and year all of type int. Accept two dates from the user and display the difference between the dates. All necessary checking must be done.

3. Raising a number n to power p is the same as multiplying n by itself p times. Write a function called power ( ) that takes a double value for n and at int value for p, and returns the result as double value. Use default argument of 2 for p, so that if this argument is omitted, the number n will be swuared. Write a main function that gets values from users to test this function.

4. Write a function that, when you call it, displays a message telling how many times it has called. I have been called 3 times, or whatever. Write a main ( ) function that calls this function in two different ways. First use an external variable to store count. Second use a local static variable. Which is more appropriate? Why cannot you use an automatic variable?

5. Write a function called swap ( ) that intercahnges two int values passed to it by the calling program. (Note that this function swaps the values of the variables in the calling program, not hots in the function). You will need to decide how to pass the arguments. Create a main ( ) program to exercise the function.

6. Write a program that uses an overloaded operator + = to concentrate two strings.

7. Create a class time. The class should possess functions for getting time, displaying time and to find the difference between any two given times.

8. Suppose a publishing company markets both books and audio cassettes. Create a class publication that stores the title and price of the publication. From this derive two classes: book which adds page count, and tape which add paying time. Each of these three classes should have get date ( ) and put data ( ) function.

9. Create an employee class to store the employee number, name and basic pay of an employee. Derive three classes Manager, Assistant an Peon from the class employee. Each of these class should store the data int percentage hra, other allowances and deductions.

Add necessary functions to the classes

1. Read required data

2. Display data

3. To calculate Gross salary

4. To calculate Net Salary

   Write a main program to test these, Modify the program 3 to add base class sales

   to store the sales amount of last three months. The class should get data ( ) and put data ( ) functions for reading and displying data respectively. After book and tape classes to give them the properties of both publication and sales. An object of class book or tape should input sale data along with other data. Wirte main function to create book and tape objects to test their capabilities.

10. Write a program to read a group of numbers from the user and places them in an array of type float. Then the program should find the average and print the result. Use pointer notation where ever possible.

11. Write a program to read a string in lower case letters and stores it in a class string data. Then use a function change case ( ) to convert the lower case letters of the string to upper case. You can use the library function to upper ( ) for this purpose.

This function takes a single character a argument and changes the character to upper case. Use pointers.

12. Make you own version of the library functions stremp (s_1, s_2) . While comparing s_1 and s_2, if s_1 comes alphabetically first then the function should return 1, if both s_1 and s_2 and s_2 are same return 0 and otherwise return -1. It should take two char string as arguments. Compare character by characters and return int. Write a main

    program to test the function.

13. Make necessary changes in the linked list program so that it displays the iterms in the order we entered. The program given in this chapter will print the data in the reverse order.

14. Create a class to hold the roll number, name and mark of 10 students. Add functions to read data, to display data and to prepare rank list based on mark. Do this program using both arrays and linked list (Use pointers wherever possible).

Practical Programs

The following 12 practical programs have to be done a computer using Turbo or Boreland C++ program.

1. Product of two matrices of any order
2. Inverse of a square matrix
3. General interactive method to solve $f(x) = 0$ by changing to the form $x = g(x)$.
4. Bisection Method
5. Newton - Raphson's method
6. Regula-Falsi method
7. Trapezoidal rule of integration
8. Simpson's one-third rule of integration
9. Simpson's three-eight rule of integration
10. Euler's method to solve a first order differential equation with a given initial condition.
11. Euler's modified method
12. Runge-Kutta method of order 4

13. Write a program to sort name list

14. Write a program to sort the number list in ascending order

15. Write a program to find the factorial of a number using the concept of function.

16. Write a program to display the name of day in a week.

17. Using function concept write a program to find the value of nC

18. Write a program using pointers to read an array of natural numbers and print its elements in reverse order.

School of Distance Education

UNIVERSITY OF KERALA

SENATE HOUSE CAMPUS, PALAYAM, TRIVANDRUM – 695 034

Phone: 0471 2300137, website: www.ideku.net

ASSIGNMENT

Course : -------------------- (Previous/Final) ----------------------------

Year of admission: ---

Paper Code : ---

Title of the Paper : ---

Name and Address of the Contact Centre

SDE Enrolment Number

Exam Register Number

Name and Address of Student

Student's Name: ---

Address: ---

Phone No: ---

Mobile : ---

Date of Submission: ______________________

FOR OFFICE USE ONLY

Marks Awarded

/ **15**

Name and Signature of the Evaluator